

# Kaluza-Klein matter states and compact extra dimensions detection

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Criteria for compact dimension detection are derived from classical and quantum motion of mass probes in 5 dimensions, confirming the higher dimensional nature of space-time within experimental accuracy.

PACS numbers: 11.10.Kk, 04.50.+h

## Introduction

The recent search for extra spatial dimensions has been performed with experiments focused on a measurement of the universal gravitational constant  $G$ , currently measuring the force of gravity at the sub-millimeter range. Such a deviation from Newton's inverse-square law, in accordance with higher dimensional gravity models, can be taken as proof of the existence of large spatial dimensions at low energy, physical interactions detected in 4-dimensional space-time, are induced by dynamic behavior of fields and physical observable quantities defined over such extra dimensions, offering the possibility for indirect detection of such dimensions within actual experimental energy range. Although

string theory postulates 26 compact extra dimensions at the Planck scale, and large compact and extended dimensions may exist, the scope of the present work focuses on the 5-dimensional nature of space-time and realistic matter states derived from the dynamics on such 5-dimensional manifold, that can be detected experimentally at current energies, despite the energy scale magnitude of the Planck sized dimensions.

The criteria for the detection of spatial compact extra dimensions, as originally postulated by O. Klein <sup>1</sup> at the Planck scale, are given by the analysis of the motion of a mass probe in 5 dimensions. As the motion occurs in 5-dimensional space-time, physical characteristics will be acquired by the mass probe, such characteristics manifested in 4-dimensional space-time must match the experimental physical measured values in laboratory, revealing the true physical nature of the higher dimensional space-time and therefore indicating the without any sign of ambiguity the presence of an extra compact dimension. The first verifiable prediction derived from this higher dimensional classical motion, indicates the existence and origin of the electric quantised charge  $q_n = ne$ ,  $n = 1, 2, 3, 4, \dots$ , in 4-dimensions, and then the corresponding quantum law of motion, the Dirac equation for massless fermions propagating in 5-dimensional space-time will yield the energy spectrum, revealing the origin and magnitude of 4-dimensional rest mass for spin  $\frac{1}{2}$  fermions within experimental laboratory accuracy. The fundamental constant of Quantum Electrodynamics  $\alpha$  is required to contribute to the Kaluza-Klein gravitational coupling constant  $\kappa$ .

### **Kaluza-Klein Space-time**

Realistic unification through the Kaluza-Klein approach requires  $d=5$  manifold topology to be:  $M^5 = M^4 \times S^1$ , the coordinate points are described by  $x^{\hat{\mu}} = (x^{\mu}, x^5)$  and the

spatial extra dimension radius  $R_5$  is of Planck length order. Finally the extra spatial dimension is required to be periodic  $x^5 = x^5 + 2\pi R_5$ . The d=5 metric of the space-time and action are:

$${}^{(5)}g_{\mu\nu} = \begin{pmatrix} {}^{(4)}g_{\mu\nu} + \kappa^2 \Phi^2 A_\mu A_\nu & \kappa \Phi^2 A_\mu \\ \kappa \Phi^2 A_\nu & \Phi^2 \end{pmatrix}, \quad {}^{(5)}S = \frac{1}{\kappa^2 2\pi R_5} \int {}^{(5)}R \sqrt{|{}^{(5)}g|} d^5x. \quad (1)$$

so far the gravitational coupling constant usually given by:

$$\kappa^2 = \frac{16\pi G}{c^4}. \quad (2)$$

### Classical 5-dimensional motion

For the mass probe 5-dimensional motion, consider the Lagrangian<sup>2</sup> :

$$L = \frac{1}{2} \left( g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu - \Phi (\dot{x}^5 + \kappa A_\mu \dot{x}^\mu)^2 \right), \quad (3)$$

the geodesic law of motion derived:

$$\ddot{x}^\mu + \Gamma_{\alpha\beta}^\mu \dot{x}^\alpha \dot{x}^\beta = -\frac{p_5}{m} \kappa F_\nu^\mu \dot{x}^\nu - \frac{1}{2} \left( \frac{p_5}{m} \right)^2 \Phi^{-2} \nabla^\mu \Phi, \quad (4)$$

corresponds to the 4-dimensional Lorentz force motion law, if the electric charge ansatz is:

$$\kappa p_5 = q. \quad (5)$$

### Alpha

The Quantum Electrodynamics fundamental constant rule definition, consistent with the 5-dimensional geometrical background and the 4-dimensional experimentally measurements obtained for alpha, requires the definition of Kaluza-Klein 5-dimensional alpha probability density distribution:

$$\alpha_5^H = \frac{v_H r_H}{2\pi v_{planck} R_5^2}, \quad (6)$$

in order to calculate the probability associated to alpha in four dimensional space-time:

$$\alpha_4 = \alpha_5 (2\pi R_5). \quad (7)$$

The accuracy of the probability value derived from the 5-d probability density distribution, is in agreement with the corresponding probability value given by the standard definition of alpha, as well as with the measurements obtained from experiment. The  $v$  vev energy scale, characterizing the 4-dimensional Standard Model spontaneous symmetry breakdown phenomenology and the Higgs mechanism, parameterizes the probability distribution and fixes the probability measure. The  $v$  vev input is set at 246 Gev, and the observed electromagnetic strength, becomes defined when the spontaneous symmetry breakdown occurs at 246 Gev. The Higgs particle estimated radius  $r_H$  magnitude is assumed to be  $5.82869 \times 10^{-21}$  meters. The 4-dimensional alpha probability definition and value  $\alpha_4 = \alpha_5^H (2\pi R_5)$ , obtained from the parameterized probability density distribution definition (6), will be retained for all motion criteria and operator definitions.

The parametric probability density definition can be validated further, if one assumes an energy scale symmetry breakdown value  $v$  of electron rest energy order, and the radius magnitude of the hypothetical associated boson to be of classical electron radius order i.e.:

$$\alpha_4 = \frac{m_e r_e}{m_{planck} R_5}. \quad (8)$$

## Electric charge spectrum

In order to identify the electric charge measured in 4-dimensional space-time, from the 5-dimensional probe motion described by (4), the alpha contribution to the gravitational coupling is:

$$\kappa_{eff}^2 = \frac{4\pi \varepsilon_0 \alpha G}{c^2}. \quad (9)$$

Taking the mass probe fifth dimensional momentum component as  $p_5 = \frac{n\hbar}{R_5}$ , the definition of the electric charge ansatz yields the quantised electron charge:

$$\kappa_{eff} p_5 = ne. \quad (10)$$

The calculated charge corresponds to the observed experimental value,

$$e \equiv 1.60217733 \times 10^{-19} \text{ coulombs}$$

Therefore the Lorentz force charge ansatz, now can be recast in operator form, given the following definition for the charge operator:

$$\hat{Q} \equiv i\kappa_{eff} \hbar \frac{\partial}{\partial x^5}, \quad (11)$$

where the charge amplitude  $\Phi_n \equiv \Phi_n(x^5) = e^{inx^5/R_5}$  satisfies:

$$\hat{Q}\Phi = q_n \Phi_n \quad n = 1, 2, 3 \dots m, \dots \quad (12)$$

Experimentally measured quantised charges, are eigenvalues of the Kaluza-Klein Charge Quantisation law and are therefore a consequence of the motion of the mass probe in 5-dimensions, being the fifth dimension a spatial compact dimension of Planck size radius.

## Quantum 5-dimensional motion

While the mass probe 5-d classical motion manifests the electric charge in 4 dimensions, the quantum motion will require a massless charged probe spin  $\frac{1}{2}$  fermion satisfying the Dirac law in 5 dimensions <sup>3</sup> :

$$D^{(4)}\psi + D^{(5)}\psi = 0, \quad (13)$$

the Dirac operator defined over the fifth dimension acting on the massless charged fermion probe wavefunction  $\psi(x, x^5) = \psi(x)\Phi_n(x^5)$  leads to the corresponding 4-dimensional energy spectrum eigenvalues:

$$D^{(5)}\psi = E_n\psi, \quad (14)$$

The 5-dimensional quantum motion of a massless charged fermion, manifests massive spin  $\frac{1}{2}$  electrons in 4-dimensional space-time for the following definition for the 5-d Dirac Operator:

$$D^{(5)} \equiv \gamma^5 c^2 \hat{M} \quad (15)$$

## The electron mass gap

The mass operator establishing an equivalence relationship between the electric charge and the electron rest mass is given by:

$$\hat{M} \equiv \frac{R_5}{r_e} \sqrt{\frac{\alpha}{4\pi G \epsilon_0}} \hat{Q}, \quad (16)$$

where  $r_e$  is the classical electron radius.

Such operator is defined over the compact dimension, acting upon the charge amplitude previously defined, defines the electron mass gap for the fifth dimension. The spectrum

values derived from the 5-dimensional quantum motion law, clearly shows that zero modes are massless and do not have electric charge, as well as the fact that the first mode  $m_1$  equates the experimentally measured value of the electron mass  $m_{oe} = 9.1093897 \times 10^{-31}$  kg.

## **Conclusion**

The classical and quantum motion of probes over the 5-dimensional space-time, predict the manifestation of quantised electric charges and the electron mass spectrum values corresponding to the experimental accuracy available in 4-dimensional space-time, electron states are a manifestation of the 5-dimensional geometry unifying gravity and electromagnetic forces. Therefore, such classical and quantum motion criteria indicate the presence of the fifth dimension as a compact extra dimension of Planck radius at each point of 4-dimensional space-time, confirming beyond any doubt the physical existence of the fifth dimension as postulated by O. Klein. The physical structure of the 5-dimensional space-time proposed by the Kaluza-Klein unification is not anymore a mere theoretical assumption, and the criteria set the first step towards string and brane theory experimental verifications.

I want to acknowledge useful conversations with G. Torres.

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<sup>1</sup> Oscar Klein, The atomicity of electricity as a quantum theory law, NATURE **118**, 516 (1926).

<sup>2</sup> Lochlain O'RaiFeartaigh, Rev. Mod. Phys. **72**, 1 (2000).

<sup>3</sup> Edward Witten, Search for a realistic Kaluza-Klein theory, Nuclear Physics **B186**, 412-428 (1981).